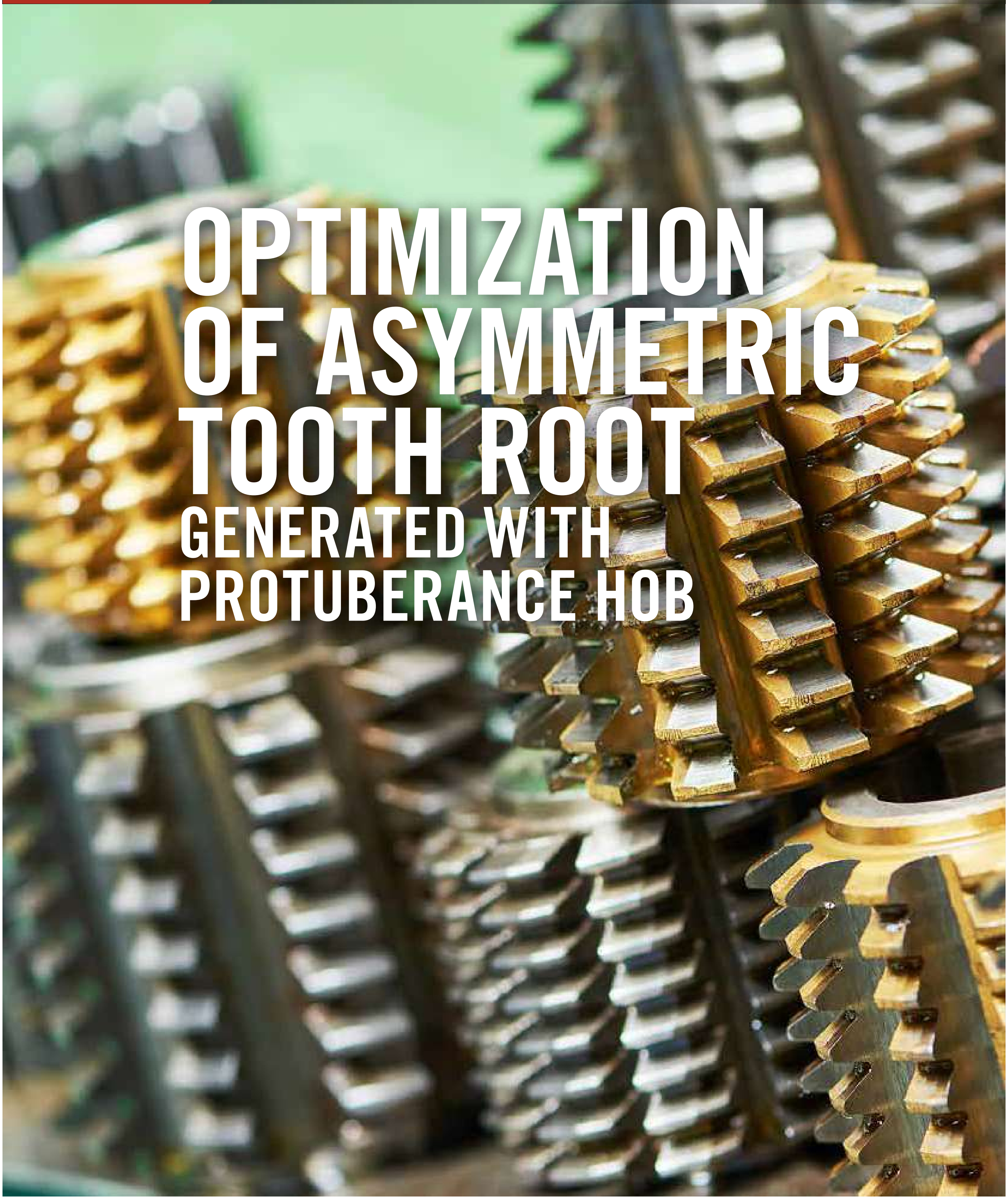


OPTIMIZATION OF ASYMMETRIC TOOTH ROOT
GENERATED WITH PROTUBERANCE HOB

Demonstrating the construction and optimization of the tooth root fillet while leaving required grinding stock on the involute tooth flanks.

By DR. A.L. KAPELEVICH and DR. Y.V. SHEKHTMAN

The Direct Gear Design [1] method optimizes various parameters and elements of gear tooth geometry to achieve the required gear-drive performance. One such critical element of the tooth profile is the root fillet. Previous publications had considered asymmetric tooth root fillet optimization assuming the tooth involute flanks and root fillets are processed (machined) simultaneously. However, there are many applications that require high gear tooth flank accuracy and high-load capacity as well as low gear production cost. In order to satisfy these requirements, the tooth involute flanks and root fillets are processed separately. The gear blank is machined by the topping protuberance hob, which finalizes the root fillet, tooth tip diameter, and chamfers but leaves stock for tooth flank grinding (Figure 1). Then, after gear heat treatment (carburizing plus case hardening) — and in some cases shot peening of the tooth root — the tooth flanks are processed by highly productive generating grinding, removing the grinding stock. As a result, the compressive residual stress in the root fillet developed during case hardening is retained. This fabrication sequence is mostly typical for automotive transmission gears.]

Since tooth root load capacity is a major contributor to gear transmission performance, the reduction of tooth bending stress concentration in asymmetric gears generated with a protuberance hob is critically important.

This paper demonstrates the construction and optimization of the tooth root fillet while leaving required grinding stock on the involute tooth flanks. It also describes a reversed generation of the protuberance hob tooth profile using the completely defined gear tooth profile, which includes the optimized root fillet and grinding stock on the tooth flanks.

Figure 1: Asymmetric gear with tooth root generated with protuberance hob.

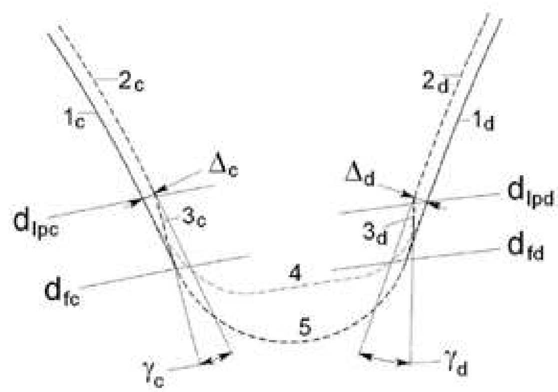


Figure 2: Initial construction of root fillet; 1_d and 1_c — drive and coast final involute flanks after grinding, 2_d and 2_c — drive and coast preliminary involute flanks before grinding, 3_d and 3_c — drive and coast interim involute fillet flanks that will be removed by tooth flank grinding, 4 – trajectory of mating gear tooth tip in zero backlash mesh; 5 — initial tooth root fillet; Δ_d and Δ_c — drive and coast flank grinding stocks; γ_d and γ_c – drive and coast flank interim fillet profile angles; d_{lpd} and d_{lpc} — drive and coast flank lowest contact point diameters; d_{fd} and d_{fc} — drive and coast flank form diameters.

1 TOOTH ROOT FILLET CONSTRUCTION AND OPTIMIZATION

Construction of the initial root fillet of the asymmetric tooth is shown in Figure 2. The direct gear design method applied to asymmetric gears [2] defines the final (after grinding) involute tooth flanks 1_d and 1_c and also the trajectory of the mating gear tooth tip in zero backlash mesh 4.

Then, the preliminary involute flanks 2_d and 2_c (after hobbing) are constructed equidistant to the final tooth flanks, providing Δ_d and Δ_c — drive and coast flank grinding stocks. The interim drive and coast fillet flanks 3_d and 3_c are for transitioning between the preliminary flanks 2_d and 2_c and the root fillet. They intersect the preliminary flanks, providing drive and coast flank grinding stocks Δ_d and Δ_c at the lowest points of contact with the mating gear tooth tip at the drive and coast flank lowest contact point diameters d_{lpd} and d_{lpc}. These interim fillet profiles are typically involute, formed by the straight protuberance profiles of the protuberance hob. The interim fillet profiles are designed to be machined by hobbing, and later removed by tooth flank grinding after heat treatment.

The initial tooth root fillet 5 is a circular arc that lays below the trajectory of the mating gear tooth tip 4 to exclude root-tip interference. It is tangential to both the interim drive and coast fillet flanks 3_d and 3_c. For practical purposes, the initial tooth root fillet 5 is also almost tangential to the final involute tooth flanks 1_d and 1_c, leaving a small amount of undercut (Figure 3) to accommodate manufacturing tolerances and some gear distortion after heat treatment and to guarantee an exit for the grinding wheel without creating a step between the ground tooth flank and hobbed root fillet.

The goal of tooth root fillet optimization is to achieve minimal stress concentration in the tooth fillet profile. As a result, the maximum bending stress is evenly distributed along a large portion of the fillet. The root fillet optimization method, developed by Dr. Y.V. Shekhtman, uses three major procedures: defining functions to approximate the fillet profile, FEA for stress calculation, and a random search algorithm to define the optimal set of coefficients for the trigonometric functions, allowing them to reach minimal bending stress.

Unlike gears with an optimized ground tooth root where the initial root fillet is the trajectory of the mating gear tooth tip in a zero-backlash mesh (item 4 in the Figure 2), gears with a tooth root generated by the protuberance hob have circular arc initial root fillets (item 5 in the Figure 2). Finite element nodes are evenly distributed along this initial root fillet profile (Figure 4a). The center of the initial root fillet is connected to these finite element nodes. The first and last finite element nodes of the initial fillet profile, located on form diameters d_{fd} and d_{fc}, cannot be moved during the optimization process. The rest of the finite element nodes are moved along straight lines perpendicular to the fillet profile. Bending stresses are calculated for every iteration of the fillet profile configuration. A new position is defined for each finite element node based on the results of the previous iterations in order to reduce stress values. The optimization process stops when the maximum stress value cannot be further reduced. The optimized fillet profile provides even stress distribution along a significant length of the stretched (Figure 4b) and compressed (Figure. 4c) portions of the root fillet.

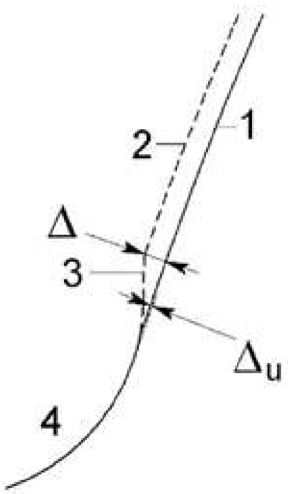


Figure 3: Small undercut D_u in the transition point between the final involute flank 1 and initial root fillet 4, 2 — preliminary involute flank before grinding, 3 — interim involute fillet flank; Δ — flank grinding stock.

Figure 5 shows a comparison of the root fillet and stress charts along the tooth profile before and after root fillet optimization.

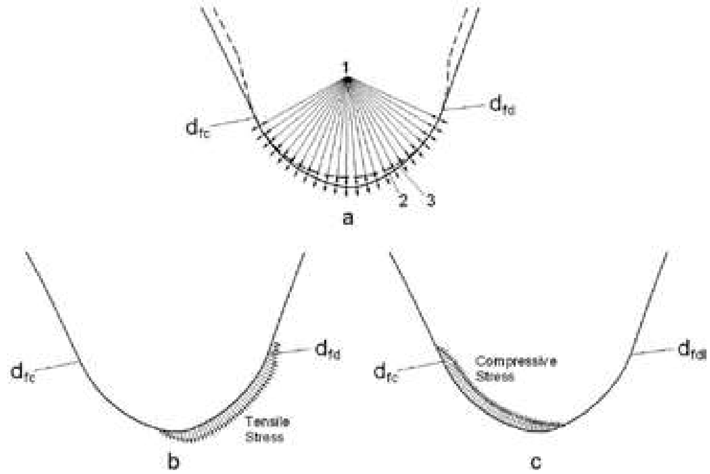


Figure 4: Tooth root fillet optimization: a — FE node movement, b — tensile stress distribution, c — compressive stress distribution; d_{fd}, d_{fc} — form diameters circles of the drive and coast tooth flanks.

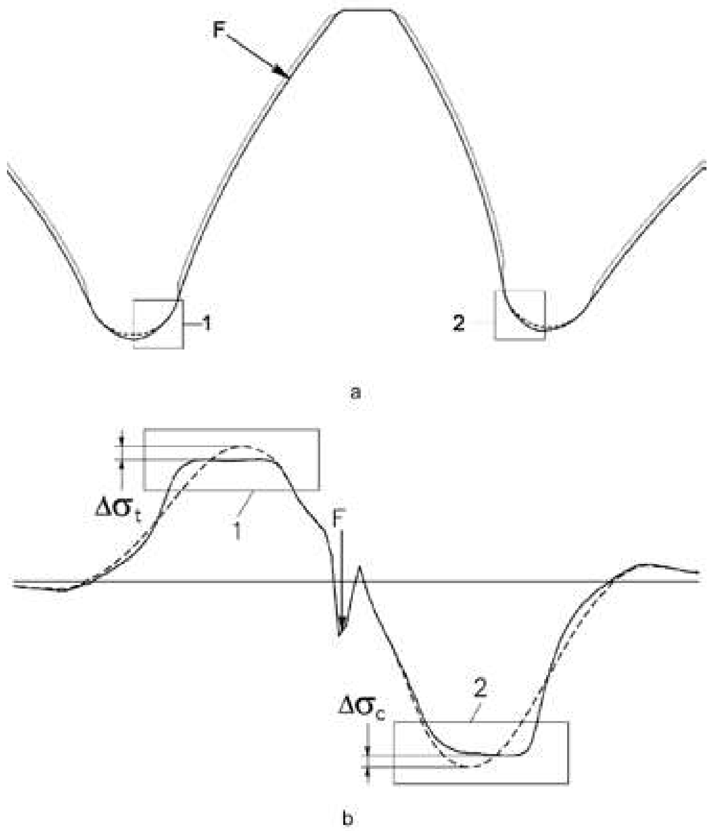


Figure 5: Root fillet optimization: a — tooth profile with the load F applied at the HPSTC, b — stress distribution along tooth profile; dashed line — initial circular root fillet and solid line — optimized root fillet; 1 — tensile stress area, 2 — compressive stress area; $\Delta\sigma_t$ — tensile stress reduction, $\Delta\sigma_c$ — compressive stress reduction.

2 REVERSED GENERATION OF TOOLING (HOB) PROFILE

The fabrication process for gears with ground involute flanks and unground root fillets usually assumes the use of a protuberance hob cutter.

In this case, the reverse generating technique is applied to find the protuberance hob rack profile using the already designed gear profile with the optimized root. This technique assumes that in the gear-rack mesh, every point of the gear tooth profile has a corresponding point on the rack tooth profile. Figure 6 demonstrates how the tooling rack profile point A_t position is defined from the gear tooth profile point A_g position. In order to find the generating rack profile point A_t corresponding to the gear tooth profile point A_g, the line A_gB_g perpendicular to the tooth profile at point A_g is constructed. Point B_g lies at the intersection of the line A_gB_g with the gear pitch circle. The gear tooth profile and line A_gB_g are rotated by angle γ_g relative to the gear center until point B_g reaches its pitch point position B_g', where gear pitch circle 3 is tangent to rack pitch line 4. Then point A_g is in position A_g', where gear tooth profile 1' is tangent to rack profile 2'. The line $A_g'B_g'$ is moved parallel to rack pitch line 4 by distance $B_g'B_t$, which is equal to the length of arc B_gB_g'. This movement puts point A_g' in position A_t along the rack

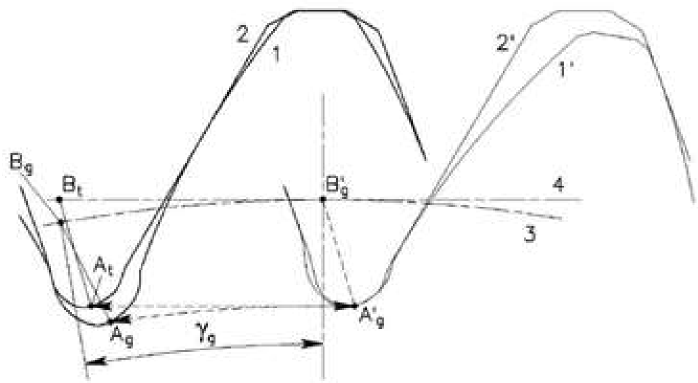


Figure 6: Reverse generating of the protuberance hob rack profile; 1 and 1' — gear profile positions; 2 and 2' — rack cutter profile positions; 3 — gear pitch circle in a mesh with the rack; 4 — rack pitch line in a mesh with the gear.

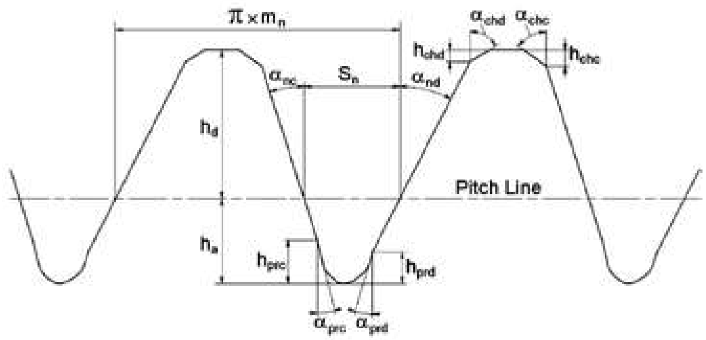


Figure 7: Protuberance hob profile parameters: m_n — normal module, S_n — normal tooth thickness at the generating pitch line, h_a — addendum, h_d — dedendum, α_{nd} and α_{nc} — drive and coast normal pressure angles, h_{prd} and h_{prc} — drive and coast protuberance heights, α_{prd} and α_{prc} — drive and coast protuberance angles, h_{chd} and h_{chc} — drive and coast chamfer heights, and α_{chd} and α_{chc} — drive and coast chamfer angles.

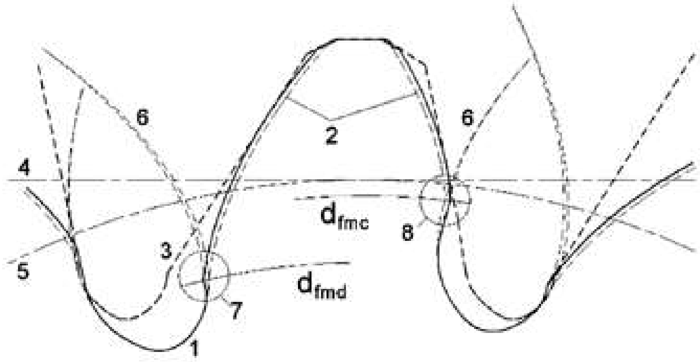


Figure 8: Gear profile undercut by protuberance hob tooth tip: 1 — machined gear profile with the grinding stock, 2 —final gear flanks profiles after grinding, 3 —protuberance hob profile, 4 —hobbing generating pitch line, 5 —hobbing generating pitch diameter, 6 —hob tooth tip trajectories, 7 and 8 — drive and coast flank undercut areas, d_{fmd} and d_{fmc} — drive and coast flank form diameters of the gear tooth with the grinding stock.

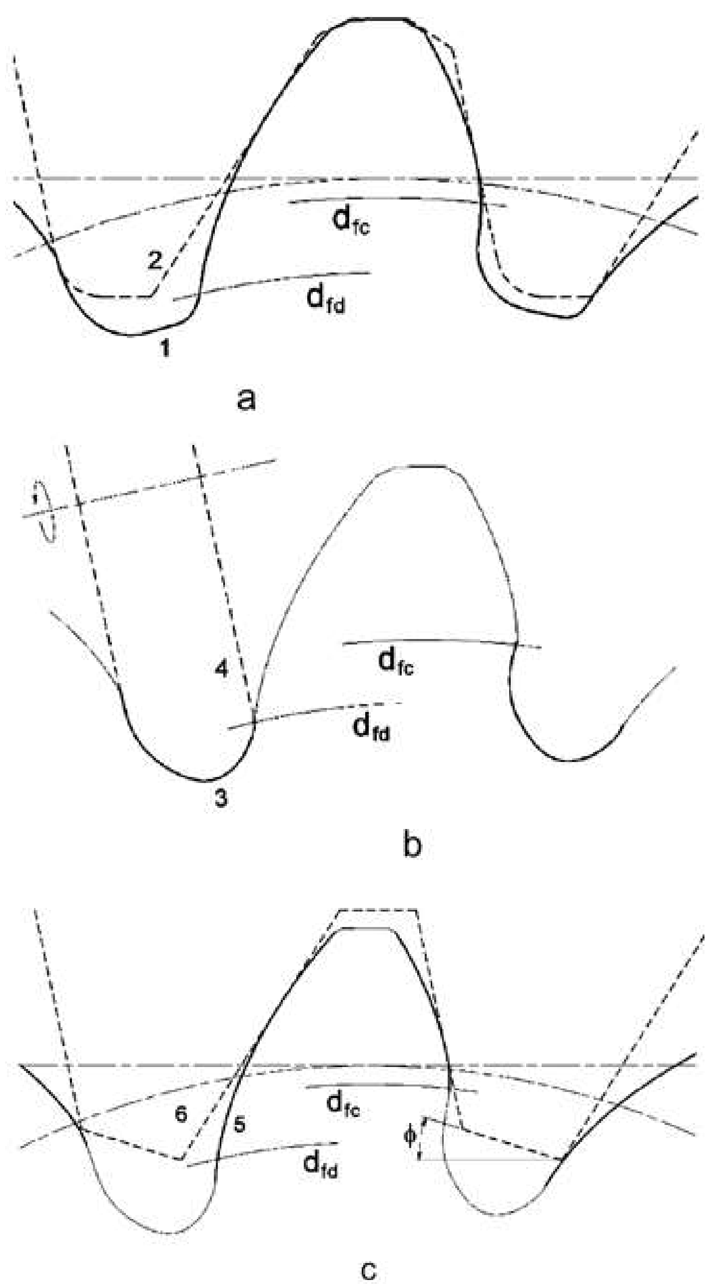
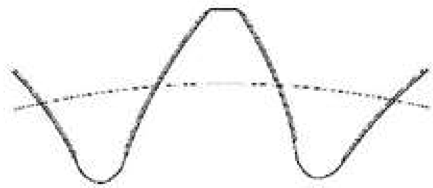
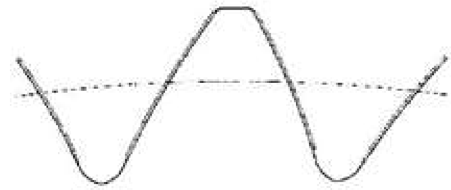


Figure 9: Processing of asymmetric gear with low number of teeth: a — preliminary gear hobbing, 1 — preliminary machined gear profile, 2 — roughing non-protuberance hob profile; b — optimized root milling, 3 — gear with machined root fillet, 4 — form disk mill cutter; c — tooth flank grinding, 5 — gear with ground tooth flanks, 6 — generating grinding wheel; ϕ — grinding wheel thread tip angle.

profile, which corresponds to point A_g on the gear tooth profile. This approach allows us to define any generating rack profile point that corresponds a specific gear tooth profile point. For helical gears, this technique is applied in the normal to the gear tooth line section. For topping protuberance hobs, the gear's outer diameter and chamfer cutting profiles are also generated.

The selection of the radial position of the pitch point B_g' (defining the gear pitch circle and rack pitch line) in the reverse generating process is very important. It affects all protuberance hob profile parameters (Figure 7). If the pitch point is located above the nominal gear pitch diameter in the mesh with the mating gear, the hob rack module m_n, α_{nd}, and α_{nc} (drive and coast pressure angles) and α_{prd} and α_{prc} (drive and coast protuberance angles) are increased. This leads to a hob protuberance size reduction that may result in increased root fillet surface roughness and excessive hob protuberance wear. If the pitch point is below the nominal gear pitch diameter in the mesh with the mating gear, the hob rack module m_n, α_{nd}, and α_{nc} (drive and coast pressure angles) and α_{prd} and α_{prc} (drive and coast protuberance angles) are reduced. This leads to a hob protuberance size increase and a smoother root fillet surface finish. However, the drive and coast protuberance angles should

Optimization of the tooth root fillet of asymmetric gears generated with a protuberance hob allows for a reduction in root tensile stress by about 10 percent compared to an asymmetric gear pair with conventional trochoidal roots generated by a full tip radius profile protuberance hob.

Gear Data				
Gear	 Pinion		 Gear	
Number of Teeth	23		41	
Normal Module, mm	3.000			
Normal Pressure Angle	30.00°/22.00°*			
Helix Angle	30.00°			
Pitch Diameter (PD), mm	79.674		142.028	
Base Diameter, mm	66.293/72.203*		118.175/128.710*	
Tooth Tip Diameter, mm	86.475		148.424	
Form Diameter, mm	73.728/74.590*		135.774/136.485*	
Root Fillet Profile	Conventional Trochoidal**	Optimized	Conventional Trochoidal**	Optimized
Root Diameter, mm	71.654	71.448	133.714	133.520
Tooth Thickness at PD, mm	4.744		4.515	
Face Width, mm	40.00		38.00	
Tooth Tip Chamfer, mm	0.20		0.20	
Transverse Pressure Angle	33.70°/25.00°*			
Transverse Contact Ratio	1.25/1.42*			
Axial Overlap	2.00			
Total Contact Ratio	3.25/3.42*			
Tooling (Topping Hob) Generating Rack Data				
Normal Grinding Stock, mm	0.100/0.100*		0.100/0.100*	
Generating Rack Module, mm	2.921		2.921	
Generating Pitch Diameter, mm	76.918		137.118	
Pressure Angle	27.20°/17.78°*		27.20°/17.78°*	
Dedendum, mm	4.778		5.653	
Protuberance Pressure Angle	16.22°/13.56°*		19.61°/13.74°*	
Root Fillet Profile	Conventional Trochoidal**	Optimized	Conventional Trochoidal**	Optimized
Addendum, mm	2.632	2.735	1.702	1.799
Protuberance Addendum, mm	0.916/1.302*	1.019/1.405*	1.062/1.444*	1.159/1.541
Rack Tooth Tip Radius, mm	0.750	N/A	0.850	N/A
Rack Chamfer Angle, mm	58.4°/54.4°*		58.0°/53.6°*	
Rack Chamfer Depth, mm	0.37/0.51*		0.35/0.48*	
Loads and Root Stresses				
Drive Flank Torque, Nm	700		1248	
Root Fillet Profile	Conventional Trochoidal**	Optimized	Conventional Trochoidal**	Optimized
Root Tensile Stress, MPa	391	351	396	356
Tensile Stress Reduction	-	10.2%	-	10.1%

* Drive/Coast Flank

**generated by a full tip radius protuberance hob

Table 1

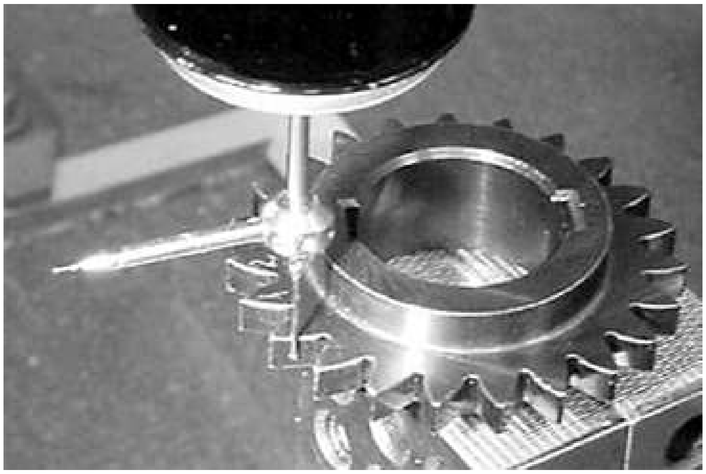


Figure 10: CMM measurement of asymmetric gear.

not be less than about 8 degrees, otherwise the back angle of the protuberance cutting edge could be too small for proper machining.

3 COMPARING GEARS WITH CONVENTIONAL TROCHOIDAL AND OPTIMIZED TOOTH ROOTS

Table 1 presents a comparison of two asymmetric gear pairs with identical flank geometry and different root fillets — a conventional trochoidal fillet generated by a protuberance hob with a full radius tooth tip, and an optimized fillet generated by a protuberance hob with a special shape tooth tip.

Under the same load, the asymmetric gear pair with the optimized root has its root tensile stress reduced by about 10 percent compared to the asymmetric gear pair with the conventional trochoidal root.

4 PROTUBERANCE HOB ROOT GENERATION LIMITATIONS

In some cases, a combination of the asymmetric gear's geometric parameters does not allow the use of a protuberance hob for preliminary machining of the tooth flanks and final machining of the root fillet. This is typical for gears with a low number of teeth and also a low coast profile pressure angle. In such cases (see Figure 8), the sweep of the protuberance hob tooth tip undercuts the tooth flanks near the drive and coast flank form diameters of the gear tooth with the grinding stock.

In this case, the gear machining process should be altered, replacing protuberance hobbing with conventional non-protuberance hobbing (Figure 9a) and optimized root milling using a mill cutter shaped as the optimized root fillet (Figure 9b). After heat treatment, the gear flanks are ground with a helical grinding wheel. Asymmetric gears have different drive and coast flank form diameters, and the helical grinding wheel thread tip should be dressed with a chamfer angle to avoid interference with the root fillet near the coast flank form diameter (Figure 9c).

Generating grinding of the asymmetric gear flank is described in [3].

5 INSPECTION OF OPTIMIZED TOOTH ROOT

The optimized tooth fillet of an asymmetric gear generated with a protuberance hob must be inspected by CMM in the same way as the involute flanks (Figure 10). Deviation of the actual root fillet profile is relative to the designed (CAD) profile at average material conditions. Root profile tolerance should be about 2-3 accuracy grades lower than the ground involute flank profile tolerance.

SUMMARY

◗ The article presents the direct gear design tooth root fillet optimization technique for asymmetric gears generated with a protuberance hob.

◗ Optimization of the tooth root fillet of asymmetric gears generated with a protuberance hob allows for a reduction in root tensile stress by about 10 percent compared to an asymmetric gear pair with conventional trochoidal roots generated by a full tip radius profile protuberance hob. Combined with other benefits of asymmetric gears [2], this root stress reduction makes asymmetric gears highly desirable for automotive transmissions.

◗ The optimized root fillet of asymmetric gears generated with a protuberance hob must be CMM inspected as thoroughly as the involute tooth flanks.

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